**Question 1**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | r | (p -> r) | (q -> r) | (p -> r) v (q -> r) |
| T | T | T | T | T | T |
| T | T | F | F | F | F |
| T | F | T | T | T | T |
| T | F | F | F | T | T |
| F | T | T | T | T | T |
| F | T | F | T | F | T |
| F | F | T | T | T | T |
| F | F | F | T | T | T |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | r | (p ^ q) | (p ^ q) -> r |
| T | T | T | T | T |
| T | T | F | T | F |
| T | F | T | F | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | T | F | F | T |
| F | F | T | F | T |
| F | F | F | F | T |

Therefore, these statements are logically equivalent

**Question 2**

1. , where the domain of x and y is all people and Faster(x, y) denotes that person x runs faster than person y.
2. , where the domain of x and y is all people and Likes(y, x) denotes that person y loves person x
3. , where the domain of x is all people and King(x) denotes that person x is a king

**Question 3**

i.e. Show that

Note:

, where and

(The Math Sorcerer, 2015)

**Question 4**

For n > 1, let P(n) denote the statement

Base step

Inductive step

For an arbitrary k > 2, assuming that P(k) is true, it remains to prove that P(k + 1), given below, holds:

Starting with the LHS of P(k + 1),

we see that the RHS of follows.

By completing the inductive step, we have proven that is true

By mathematical induction, we have also proven that for any, the statement is true

i.e. We have proven that the implication is true

(Rosen 2007)

**Question 5**

1. …
2. …

**Question 6**

Definition: A relation R on a set S is called an equivalence relation if it is reflexive, symmetric and transitive

1. **Reflexivity**

i.e. Show that

Since is reflexive

**Symmetry**

i.e. Show that

Since is symmetric

**Transitivity**

i.e. Show that

Since is transitive

Since is reflexive, symmetric and transitive, it is an equivalence relation.  
  
(The Math Sorcerer, 2018)

1. (Rosen, 2007)

**Question 7**

1. …
2. …

**Question 8**

1. Firstly, we need to select 3 students. The number of ways in which we can select 3 students from the original 9 is given by

Next, we need to select 2 staff members. The number of ways in which we can 2 staff members from the original 6 is given by

Consequently, the number of ways is given by

1. …
2. …
3. …

**Question 9**

…

**Question 10**

1. for   
   (Rosen 2007)
2. , and

(Rosen 2007)

|  |  |
| --- | --- |
|  |  |
| 0 | 1 |
| 1 | 1 |
| 2 | 2 |
| 3 | 4 |
| 4 | 7 |
| 5 | 13 |
| 6 | 24 |

(Rosen 2007)

**Question 11**

1. Suppose that a given bit string is *valid* if it contains the substring 10 and *invalid* otherwise. Moreover, suppose that is a string of length n – 1.  
     
   If s is valid, we have two possible options:
2. We can append a 0
3. We can append a 1

Either way, seeing as the string was already valid, no change to its validity will occur with either option; this accounts for good strings of length .  
  
If s is invalid, we can only form a valid string by appending one more bit, is for the final bit of s to be 1 and then for us to append a 0. Conclusively, this scenario will grant us one valid string for every invalid string with a length of that concludes with a 1. Seeing as there are strings of length and of them are valid, there will be a total of strings that are invalid. The only invalid string of length ends in 0, shown below:  
  
000…000,

so there are invalid strings with a length of which can be modified into a valid string with a length of n

Thus, the total number of valid strings is given by:

(foobar512 2016)

1. , and

|  |  |
| --- | --- |
|  |  |
| 0 | 0 |
| 1 | 0 |
| 2 | 1 |
| 3 | 4 |
| 4 | 11 |
| 5 | 26 |
| 6 | 57 |

(foobar512 2016)

**Question 12**

1. Euler Circuit  
   No, as not every vertex has even degree. e.g. deg(a) = 3  
   (Liau 2020)

Euler Path  
Yes, a, b, e, d, a, c, d

(Liau 2020)

1. Hamilton Circuit

Yes, a, b, e, d, c, a  
(Liau 2020)  
Hamilton Path

Yes, a, b, e, d, c  
(Liau 2020)

**Question 13**

1. …
2. …
3. …

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